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On a Projection Step by Generalized Orthogonal Complement Matrices for Adaptive Subspace Tracking

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Abstract Nguyen, Takahashi and Yamada [NTY'12] proposed a scheme for solving generalized Hermitian eigenvalue problem (GHEP) based on nested orthogonal complement structure. In this paper, we first point out that the previous estimate is not used properly in the time varying coordinate system in [NTY'12] for estimation of the non-first eigenvectors. For the proper use, we propose a projection step by introducing further multiplications of orthogonal complement matrices. Numerical experiments show that the convergence of non-first generalized eigenvectors become stable by the modification.

1 Introduction

Generalized Hermitian eigenvalue problem (GHEP) is an estimation problem, for a pair of Hermitian positive definite matrices $(R_y, R_x) \in \mathbb{C}^{N\times N} \times \mathbb{C}^{N\times N}$, of vectors $\boldsymbol{v}_i \in \mathbb{C}^N \setminus \{\boldsymbol{0}\}$ satisfying

$$R_y \boldsymbol{v}_i = \lambda_i R_x \boldsymbol{v}_i \quad \text{s.t. } \boldsymbol{v}_i^H R_x \boldsymbol{v}_j = \delta_{i,j} \ (i,j = 1,2,\ldots,N), \tag{1}$$

where λ_i $(0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N)$ is called the ith minor generalized eigenvalue, $(\cdot)^H$ stands for the conjugate transpose of a vector or a matrix, and $\delta_{i,j}$ is the Kronecker delta function. In this paper, (R_y, R_x) and $\boldsymbol{v}_i$ are respectively called a matrix pencil and the ith minor generalized eigenvector of (R_y, R_x). The GHEP has been attracting great attention in many branches of signal processing, e.g., subspace tracking [1], [2], blind source separation [3], fault detection [4], pattern recognition [5], and array signal processing [6]–[10].

Adaptive estimation of $\boldsymbol{v}_i$ is also required and some adaptive estimators are proposed [11]–[13]. In our previous work, we also proposed the scheme for tracking the first r minor (or principal) generalized eigenvectors, which achieved the lower estimation error and the robustness against additive noise compared with [11]–[13] (see [14, Figs. 1 and 3]) while keeping the orthogonality. This scheme reduced estimation problem of $\boldsymbol{v}_i$ $(i = 1,\ldots,r)$ of (R_y, R_x) to that of the first minor generalized eigenvector $\boldsymbol{v}_1^{(i)} \in \mathbb{C}^{N-i+1}$ of a certain smaller matrix pencil $(R_y^{(i)}, R_x^{(i)})$ $(i = 1,\ldots,r)$ by using the nested orthogonal complement structure (Section 2.1). The ith minor generalized eigenvector $\boldsymbol{v}_i$ can be calculated from $\boldsymbol{v}_1^{(i)}$ because $\boldsymbol{v}_1^{(i)}$ is an expression of $\boldsymbol{v}_i$ in a certain coordinate system $\perp_i \in \mathbb{C}^{N\times(N-i+1)}$, i.e., $\boldsymbol{v}_i = \perp_i \boldsymbol{v}_1^{(i)}$. In application to adaptive estimation, this relation is extended with the use of the time-varying coordinate system $\perp_i(k)$. However, we recently found that the scheme [14] sometimes causes numerical instability in certain situations.

In this paper, we analyse the reason of such instability and propose an effective way to resolve this issue. Since the scheme [14] assumed implicitly that the time-varying coordinate system changes smoothly, the expression in the previous coordinate system is reused in the current coordinate system. However, if the time-varying coordinate system changes relatively fast, then the consistency between the previous expression and the current coordinate system is lost, which leads to the numerical instability. To cope with this significant change of the coordinate system, we propose to use the expression of the nearest vector from the previous estimate in the current coordinate system. This modification can be done with low computational complexity just by multiplying orthogonal complement matrices. Numerical experiments in a scenario of adaptive subspace tracking show the excellent performance of the proposed stabilization.

2 Preliminaries

Let $\mathbb{R}$ and $\mathbb{C}$ be respectively the set of all real numbers and complex numbers. Capital and bold face small letters respectively express a matrix and a vector. The Euclidean norm and the B-norm of $\boldsymbol{x} := (x_1, x_2, \ldots, x_n)^T \in \mathbb{C}^N$ are respectively defined as $\|\boldsymbol{x}\| := \sqrt{\sum_{i=1}^n |x_i|^2}$ and $\|\boldsymbol{x}\|_B := \sqrt{\boldsymbol{x}^H B \boldsymbol{x}}$, where $B \in \mathbb{C}^{N\times N}$ is a Hermitian positive definite matrix and $(\cdot)^T$ stands for the transpose.

2.1 Nested Orthogonal Complement Structure

Definition 1 (B-orthogonal complement matrix) *Let $B \in \mathbb{C}^{N\times N}$ be a Hermitian positive definite matrix. For $\boldsymbol{w} \in \mathbb{C}^N \setminus \{\boldsymbol{0}\}$, $W_\perp \in \mathbb{C}^{N\times(N-1)}$ is called a B-orthogonal complement matrix of $\boldsymbol{w}$ if*

$$W_\perp^H B \boldsymbol{w} = \boldsymbol{0} \ \text{ and } \ W_\perp^H W_\perp = I_{N-1}, \tag{2}$$

where $I_{N-1} \in \mathbb{R}^{(N-1)\times(N-1)}$ is the identity matrix.

We can reduce the estimation problem of ith minor generalized eigenvector $\boldsymbol{u}_i^{(1)} \in \mathbb{C}^N$ $(i = 1,\ldots,N)$ of a matrix pencil $(A, B) \in \mathbb{C}^{N\times N} \times \mathbb{C}^{N\times N}$ to that of the first minor generalized eigenvector $\boldsymbol{u}_1^{(i)} \in \mathbb{C}^{N-i+1}$ a of certain smaller matrix pencil $(A^{(i)}, B^{(i)})$ $(i = 1,\ldots N)$ by using Fact 1 below.

Fact 1 (Nested orthogonal complement structure) *Define N matrix pencils $(A^{(i)}, B^{(i)})$ recursively as $(A^{(1)}, B^{(1)}) := (A, B)$ and*

$$\begin{cases} A^{(i+1)} := \left(U_\perp^{(i)}\right)^H A^{(i)} U_\perp^{(i)} \in \mathbb{C}^{(N-i)\times(N-i)} \\ B^{(i+1)} := \left(U_\perp^{(i)}\right)^H B^{(i)} U_\perp^{(i)} \in \mathbb{C}^{(N-i)\times(N-i)} \end{cases} \quad (i = 1,2,\ldots,N-1),$$

where $U_\perp^{(i)}$ is a $B^{(i)}$-orthogonal complement matrix of the first minor generalized eigenvector $\boldsymbol{u}_1^{(i)}$ of $(A^{(i)}, B^{(i)})$. Then the ith minor generalized eigenvector $\boldsymbol{u}_i^{(1)}$ of (A, B) ($i = 2, \ldots, N$) is expressed as

$$\boldsymbol{u}_i^{(1)} = U_\perp^{(1)} U_\perp^{(2)} \cdots U_\perp^{(i-1)} \boldsymbol{u}_1^{(i)} =: \perp_i \boldsymbol{u}_1^{(i)}. \qquad (3)$$

In (3), we can regard $\boldsymbol{u}_1^{(i)} \in \mathbb{C}^{N-i+1}$ as an expression of $\boldsymbol{u}_i^{(1)} \in \mathbb{C}^N$ in the coordinate system $\perp_i \in \mathbb{C}^{N\times(N-i+1)}$.

By combining Fact 1 with Algorithm $\mathcal{X}$ which estimates the first minor generalized eigenvector, we established the following scheme for estimation of the first r minor generalized eigenvectors [14, Scheme 1].

2.2 Adaptive estimation of generalized eigenvectors

In many signal processing applications, the matrix pencil (R_y, R_x) in (1) are defined as a pair of covariance matrices of input sequences $(\boldsymbol{y}(k))_{k\geq 0}$ and $(\boldsymbol{x}(k))_{k\geq 0}$, where k denotes discrete time index. In adaptive case, we have to estimate the matrix pencil (R_y, R_x) and its generalized eigenvectors simultaneously. For such applications, we proposed following scheme [14] where $\boldsymbol{w}_i^{(1)}(k)$ is the estimate of the ith minor generalized eigenvector $\boldsymbol{v}_i$ at time k.

Scheme 1 (Adaptive extraction of the first r minor generalized eigenvectors)

1. *Update the estimate $(R_y^{(1)}(k), R_x^{(1)}(k))$ of (R_y, R_x).*
2. *For $i = 1, \ldots, r$,*
   a. *Update the estimate $\boldsymbol{w}_1^{(i)}(k)$, from $\boldsymbol{w}_1^{(i)}(k-1)$, of the first minor generalized eigenvector of the matrix pencil $(R_y^{(i)}(k), R_x^{(i)}(k))$ on the basis of Algorithm $\mathcal{X}$.*
   b. *Compute a $R_x^{(i)}(k)$-orthogonal complement matrix $W_\perp^{(i)}(k)$ of $\boldsymbol{w}_1^{(i)}(k)$.*
   c. *Set $R_y^{(i+1)}(k) = \left(W_\perp^{(i)}(k)\right)^H R_y^{(i)}(k) W_\perp^{(i)}(k)$, $R_x^{(i+1)}(k) = \left(W_\perp^{(i)}(k)\right)^H R_x^{(i)}(k) W_\perp^{(i)}(k)$.*
3. *For $i = 2, \ldots, r$, compute*
$$\boldsymbol{w}_i^{(1)}(k) = \left(\prod_{s=1}^{i-1} W_\perp^{(s)}(k)\right) \boldsymbol{w}_1^{(i)}(k).$$
4. $k \leftarrow k+1$. *Repeat steps 1–4 until $\boldsymbol{w}_i^{(1)}(k)$ ($i = 1, \ldots, r$) converge.*

In [14], $R_x^{(i)}(k)$-orthogonal complement matrix $W_\perp^{(i)}(k)$ in Scheme 1-2.(b) is calculated as

$$W_\perp^{(i)}(k) = \begin{bmatrix} I_{N-i} - \frac{1}{1+|\bar{w}_{\mathrm{low}}^{(i)}(k)|} \bar{\boldsymbol{w}}_{\mathrm{up}}^{(i)}(k) \left(\bar{\boldsymbol{w}}_{\mathrm{up}}^{(i)}(k)\right)^H \\ -\theta\left(\bar{w}_{\mathrm{low}}^{(i)}(k)\right) \left(\bar{\boldsymbol{w}}_{\mathrm{up}}^{(i)}(k)\right)^H \end{bmatrix}, \qquad (4)$$

where $\bar{\boldsymbol{w}}_{\mathrm{up}}^{(i)}(k) \in \mathbb{C}^{N-i}$ and $\bar{w}_{\mathrm{low}}^{(i)}(k) \in \mathbb{C}$ satisfy $\bar{\boldsymbol{w}}_1^{(i)}(k) := \frac{R_x^{(i)}(k)\boldsymbol{w}_1^{(i)}(k)}{\|R_x^{(i)}(k)\boldsymbol{w}_1^{(i)}(k)\|} = [(\bar{\boldsymbol{w}}_{\mathrm{up}}^{(i)}(k))^T, \bar{w}_{\mathrm{low}}^{(i)}(k)]^T$ and $\theta : \mathbb{C} \to \mathbb{C}$ is defined as

$$\theta(\bar{w}_{\mathrm{low}}^{(i)}(k)) := \begin{cases} 1, & \text{if } \bar{w}_{\mathrm{low}}^{(i)}(k) = 0; \\ \bar{w}_{\mathrm{low}}^{(i)}(k)/|\bar{w}_{\mathrm{low}}^{(i)}(k)|, & \text{otherwise.} \end{cases} \qquad (5)$$

Moreover, examples of $(R_y^{(1)}(k), R_x^{(1)}(k))$ and Algorithm $\mathcal{X}$ are shown in Section 4 (see (11) and Algorithm 1).

3 Proper use of the previous estimate by a projection

We find that Scheme 1 has certain instability in the estimation of the non-first minor generalized eigenvectors $\boldsymbol{v}_i$ ($i = 2, \ldots, r$). In Scheme 1, since the non-first minor generalized eigenvector is estimated by using the time-varying coordinate system

$$\perp_i(k) := \prod_{s=1}^{i-1} W_\perp^{(s)}(k) \qquad (6)$$

as $\boldsymbol{w}_i^{(1)}(k) = \perp_i(k)\boldsymbol{w}_1^{(i)}(k)$, the significant change of $\perp_i(k)$ (e.g. caused by the discontinuity of θ (5)) directly influences the estimation accuracy of $\boldsymbol{v}_i$. To see this, we focus on Scheme 1-2a. In this step, the expression $\boldsymbol{w}_1^{(i)}(k)$ in the current coordinate system $\perp_i(k)$ is updated from $\boldsymbol{w}_1^{(i)}(k-1)$. However, the previous expression $\boldsymbol{w}_1^{(i)}(k-1)$ in the current coordinate system $\perp_i(k)$ does not stand for the previous estimate precisely when the significant changes of time-varying coordinate system occurs. In other words, Scheme 1 disposes the previous estimate in such a case.

We stabilize Scheme 1 by maximumly utilizing the information of the previous estimate $\boldsymbol{w}_i^{(1)}(k-1) = \perp_i(k-1)\boldsymbol{w}_1^{(i)}(k-1)$ for the update of $\boldsymbol{w}_1^{(i)}(k)$. To take over previous estimate properly, we take a projection of $\boldsymbol{w}_i^{(1)}(k-1)$ onto the current coordinate system $\perp_i(k)$, i.e.,

$$\begin{aligned} &\underset{\boldsymbol{w}\in\mathcal{R}(\perp_i(k))}{\arg\min} \left\|\boldsymbol{w}_i^{(1)}(k-1) - \boldsymbol{w}\right\|^2 \\ &= \perp_i(k)\left((\perp_i(k))^H \perp_i(k)\right)^{-1} (\perp_i(k))^H \boldsymbol{w}_i^{(1)}(k-1) \qquad (7) \\ &= \perp_i(k)(\perp_i(k))^H \boldsymbol{w}_i^{(1)}(k-1), \qquad (8) \end{aligned}$$

where $\mathcal{R}(\perp_i(k))$ stands for the range space of $\perp_i(k)$ and (8) is derived from (2) and (6). Since $\boldsymbol{w}_i^{(1)}(k-1) = \perp_i(k-1)\boldsymbol{w}_1^{(i)}(k-1) \approx \perp_i(k)(\perp_i(k))^H \boldsymbol{w}_i^{(1)}(k-1)$, it is stable to update $\boldsymbol{w}_1^{(i)}(k)$ from $(\perp_i(k))^H \boldsymbol{w}_i^{(1)}(k-1)$ in stead of $\boldsymbol{w}_1^{(i)}(k-1)$. More precisely, we propose to update $\boldsymbol{w}_1^{(i)}(k)$ from a $R_x^{(i)}(k)$-normalized vector

$$\widetilde{\boldsymbol{w}}_1^{(i)}(k-1) := \frac{(\perp_i(k))^H \boldsymbol{w}_i^{(1)}(k-1)}{\left\|(\perp_i(k))^H \boldsymbol{w}_i^{(1)}(k-1)\right\|_{R_x^{(i)}(k)}} \qquad (9)$$

instead of $\boldsymbol{w}_1^{(i)}(k-1)$, and we propose the following scheme.

Scheme 2 (Proposed scheme for stable adaptive estimation)

1. *Update the estimate $(R_y^{(1)}(k), R_x^{(1)}(k))$ of (R_y, R_x).*
2. *For $i = 1, \ldots, r$,*
   a. *If $i \geq 2$, compute $\widetilde{\boldsymbol{w}}_1^{(i)}(k-1)$ by (9).*
   b. *Update the estimate $\boldsymbol{w}_1^{(i)}(k)$, from $\widetilde{\boldsymbol{w}}_1^{(i)}(k-1)$, of the first minor generalized eigenvector of the matrix pencil $(R_y^{(i)}(k), R_x^{(i)}(k))$ on the basis of Algorithm $\mathcal{X}$.*
   c. *Compute a $R_x^{(i)}(k)$-orthogonal complement matrix $W_\perp^{(i)}(k)$ of $\boldsymbol{w}_1^{(i)}(k)$.*
   d. *Set $R_y^{(i+1)}(k) = \left(W_\perp^{(i)}(k)\right)^H R_y^{(i)}(k) W_\perp^{(i)}(k)$, $R_x^{(i+1)}(k) = \left(W_\perp^{(i)}(k)\right)^H R_x^{(i)}(k) W_\perp^{(i)}(k)$.*
3. *For $i = 2, \ldots, r$, compute*
$$\boldsymbol{w}_i^{(1)}(k) = \left(\prod_{s=1}^{i-1} W_\perp^{(s)}(k)\right) \boldsymbol{w}_1^{(i)}(k).$$

Algorithm 1 Power method based algorithm [14]

With an arbitrary initial R_x-normalized vector $\boldsymbol{w}_0 \in \mathbb{C}^N$, generate a sequence $\{\boldsymbol{w}(k)\}_{k\geq 0}$ by

$$\widehat{\boldsymbol{w}}(k+1) = \left(\frac{1}{\mu} I_N - R_x^{-1} R_y\right) \boldsymbol{w}(k)$$
$$\boldsymbol{w}(k+1) = \frac{\widehat{\boldsymbol{w}}(k+1)}{\|\widehat{\boldsymbol{w}}(k+1)\|_{R_x}} \tag{10}$$

with step size $\mu > 0$, where $\boldsymbol{w}(k)$ is the estimate of the first minor generalized eigenvector at time k.

4. $k \leftarrow k+1$. *Repeat steps 1–4 until* $\boldsymbol{w}_i^{(1)}(k)$ $(i = 1, \ldots, r)$ *converge.*

The complexity of Scheme 2 does not increase so much compared with Scheme 1. To see this, we shall observe the complexity of Scheme 2-2a. The calculation of (9) can be separated into two steps, $(\perp_i(k))^H \boldsymbol{w}_i^{(1)}(k-1)$ and $R_x^{(i)}(k)$-normalization. From (6), $(\perp_i(k))^H \boldsymbol{w}_i^{(1)}(k-1)$ requires $(i-1)$ times multiplications of vectors and orthogonal complement matrices. Since multiplication of a vector $\boldsymbol{t} \in \mathbb{C}^{N-s+1}$ and a conjugate transpose of an orthogonal complement matrix $(W_\perp^{(s)})^H \in \mathbb{C}^{(N-s)\times(N-s+1)}$ can be computed with $\mathcal{O}(2(N-s))$ multications by (4), $(\perp_i(k))^H \boldsymbol{w}_i^{(1)}(k-1)$ requires $\mathcal{O}(2Ni - i^2)$ multiplications. The $R_x^{(i)}(k)$-normalization needs $\mathcal{O}(N^2 - 2Ni + i^2)$, then the complexity of (9) is $\mathcal{O}(N^2)$. Since (9) is calculated for $i = 2, \ldots, r$, the total computational complexity of Scheme 2-2a is $\mathcal{O}(N^2 r)$. This computational complexity is much smaller than the total computational complexity $\mathcal{O}(12N^2 r - 11Nr^2 + 11r^3/6)$ of Scheme 1 [14].

4 Application to Subspace Tracking

In Scheme 1 and Scheme 2, we estimate the matrix pencil (R_y, R_x) as a pair of exponential weighted sample covariance matrices

$$\begin{cases} R_y^{(1)}(k) = \beta R_y^{(1)}(k-1) + \boldsymbol{y}(k)\boldsymbol{y}(k)^H \\ R_x^{(1)}(k) = \alpha R_x^{(1)}(k-1) + \boldsymbol{x}(k)\boldsymbol{x}(k)^H \end{cases} \tag{11}$$

with forgetting factors $\alpha, \beta \in (0,1)$. In addition, we employ Algorithm 1 (Power method based algorithm [14]) for Algorithm $\mathcal{X}$. [1]

4.1 Performance criteria

In a scenario of application to subspace tracking, we evaluate the performance of Scheme 2 (proposed scheme) compared with Scheme 1 through L $(= 100)$ independent runs. For comparison, we observe the similarity between $\boldsymbol{v}_i$ and $\boldsymbol{w}_{i,j}^{(1)}(k)$ (the ith minor generalized eigenvector of (R_y, R_x) and the estimate of $\boldsymbol{v}_i$ at time k in the jth independent run) in terms of Direction Cosine and its average

$$\mathrm{DC}_{i,j}(k) := \frac{|(\boldsymbol{w}_{i,j}^{(1)}(k))^H \boldsymbol{v}_i|}{\|\boldsymbol{w}_{i,j}^{(1)}(k)\|\|\boldsymbol{v}_i\|}$$

and

$$\mathrm{ADC}_j(k) := \frac{1}{r}\sum_{i=1}^{r} \mathrm{DC}_{i,j}(k).$$

Define the averages of $\mathrm{DC}_{i,j}(k)$ and $\mathrm{ADC}_j(k)$ in L independent runs as $\overline{\mathrm{DC}}_i(k) := \frac{1}{L}\sum_{j=1}^{L} \mathrm{DC}_{i,j}(k)$ and $\overline{\mathrm{ADC}}(k) := \frac{1}{L}\sum_{j=1}^{L} \mathrm{ADC}_j(k)$. We also measure numerical stabilities by two kinds of Sample Standard Deviations

$$\mathrm{SSD}_i(k) := \sqrt{\frac{1}{L-1}\sum_{j=1}^{L}\left(\mathrm{DC}_{i,j}(k) - \overline{\mathrm{DC}}_i(k)\right)^2}$$

and

$$\mathrm{SSD}(k) := \sqrt{\frac{1}{L-1}\sum_{j=1}^{L}\left(\mathrm{ADC}_j(k) - \overline{\mathrm{ADC}}(k)\right)^2}.$$

4.2 Numerical Experiments

The input samples are generated by

$$y(k) = \sqrt{2}\sin(0.37\pi k + \theta_1) + n_1(k)$$

and

$$x(k) = \sqrt{2}\sin(0.42\pi k + \theta_2) + \sqrt{2}\sin(0.65\pi k + \theta_3) + n_2(k),$$

where the initial phase $\theta_i (i = 1,2,3)$ has the uniform distribution in $[0, 2\pi]$, $n_1(k)$ and $n_2(k)$ are white Gaussian noise with variance $\sigma^2 = 0.1$. The input vectors $\boldsymbol{y}(k) \in \mathbb{R}^N$ and $\boldsymbol{x}(k) \in \mathbb{R}^N$ $(N = 8)$ are defined as $\boldsymbol{y}(k) := (y(k), y(k-1), \ldots, y(k-N+1))^T$ and $\boldsymbol{x}(k) := (x(k), x(k-1), \ldots, x(k-N+1))^T$ $(k \geq N)$.[2]

We adaptively estimate the first four $(r = 4)$ minor generalized eigenvectors $\boldsymbol{v}_i$ $(i = 1,2,3,4)$ of the matrix pencil (R_y, R_x) with the parameters $\alpha = \beta = 0.998$, $\eta = 1/(\lambda_N + \lambda_1)$, and the initial estimates $R_y^{(1)}(0) = R_x^{(1)}(0) = I_N$.

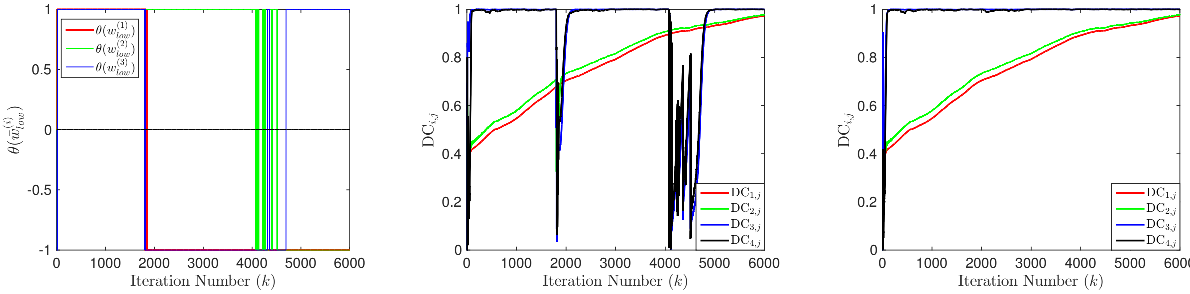
Figure 1 shows one of the outcomes in L $(= 100)$ independent runs. Figure 1(a) depicts $\theta(\bar{w}_{\mathrm{low}}^{(i)})$ $(i = 1,2,3)$. In this case, $\theta(\bar{w}_{\mathrm{low}}^{(i)})$ returns the sign of $\bar{w}_{\mathrm{low}}^{(i)}$. Figures 1(b) and 1(c) respectively depict $\mathrm{DC}_{i,j}$ $(i = 1,2,3,4)$ of Scheme 1 and Scheme 2. From Fig. 1, we observe that when $\theta(\bar{w}_{\mathrm{low}}^{(i)})$ changes significantly (i.e., when the sign of $\bar{w}_{\mathrm{low}}^{(i)}$ changes from positive/negative to negative/positive), the estimation by Scheme 1 is unstable while the estimation by Scheme 2 is stable. Figure 2 shows the result in L (=100) independent runs. Figures 2(a), 2(b) and 2(c) respectively depict $\overline{\mathrm{ADC}}$, SSD_i and SSD of Scheme 1 and Scheme 2 (note that in Fig. 2(b), there is no difference between Scheme 1 and Scheme 2 for the estimation of the first minor generalized eigenvector $\boldsymbol{v}_1$). From these figures, Scheme 2 achieves the improvement for SSD and SSD_i. Especially, SSD_i $(i = 3,4)$ in Fig. 2(b) are significantly improved.

[1] We also employ Normalized Quasi-Newton Algorithm [16] for Algorithm $\mathcal{X}$, and its performance is submitted to IEEE ICASSP 2016.

[2] The covariance matrices $R_y, R_x \in \mathbb{R}^{N\times N}$ are given as

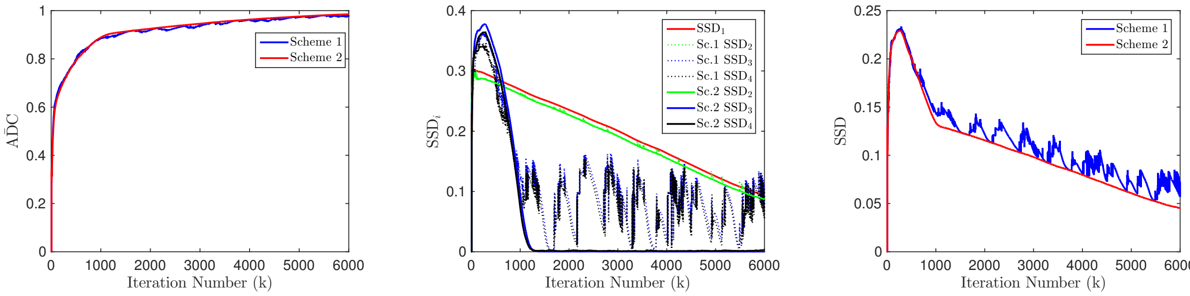
$$\begin{cases} (R_y)_{i,j} := \cos(0.37\pi(j-i)) + \delta_{i,j}\sigma^2, \\ (R_x)_{i,j} := \cos(0.42\pi(j-i)) + \cos(0.65\pi(j-i)) + \delta_{i,j}\sigma^2, \end{cases}$$

where $(\cdot)_{i,j}$ stands for the (i,j)-component of the matrix. These matrices are used for computing true generalized eigenvectors $\boldsymbol{v}_i$.



(a) $\theta(\bar{w}_{low}^{(i)})$ (b) $\mathrm{DC}_{i,j}$ of Scheme 1 (c) $\mathrm{DC}_{i,j}$ of Scheme 2

Figure 1: One of the outcomes in 100 independent runs.



(a) $\overline{\mathrm{ADC}}$ (b) SSD_i (c) SSD

Figure 2: Results in 100 independent runs (Sc.1 and Sc.2 respectively stand for Scheme 1 and Scheme 2 in (b)).

5 Conclusions

We found that the numerical instability of the original adaptive eigenvector extraction (Scheme 1) is caused by the significant change of the time-varying coordinate systems. To cope with the significant change, we proposed to use the expression of the nearest vector from the previous estimate in the current coordinate system. This proposed stabilization is realized by the projection of the previous estimate onto the range space of the current coordinate system, and its complexity is low compared with the total complexity of the original scheme. Numerical experiments show that proposed scheme achieved the excellent performance of the proposed stabilization.

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